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# General performance characteristics of an irreversible vapour absorption refrigeration system using finite time thermodynamic approach

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#### Abstract

This communication presents finite time thermodynamic analysis of vapour absorption refrigeration system considering both external as well as internal irreversibility. System is considered as consisting of an irreversible heat engine between the generator and absorber and an irreversible refrigerator between the evaporator and condenser. Optimization is made w.r.t. source/sink side heat exchanger areas, source/sink side heat transfer time and heat engine cycle and refrigeration cycle time. An optimal relation between overall coefficient of performance and maximum cooling load is obtained. Two internal irreversibility parameters (say,  $R_{\Delta S}$  is for generator–absorber assembly and  $R'_{\Delta S}$  is for evaporator–condenser assembly) have been introduced in the analysis, having fractional value for irreversible (real) system while  $R_{\Delta S} = R'_{\Delta S} = 1.0$  corresponds to endoreversible system. A detailed parametric study shows that internal irreversibility parameters have more drastic effect on performance reduction than any other external irreversibility parameter. It is also found that internal irreversibility parameters are being dominant factor for performance reduction but out of these two parameters, internal irreversibility parameter of generator–absorber assembly is more sensitive for performance reduction than the internal irreversibility parameter of evaporator–condenser assembly. © 2004 Elsevier SAS. All rights reserved.

Keywords: Finite time thermodynamics; Vapour absorption refrigeration system; External and internal irreversibility; Optimal area distribution; Optimal cycle time distribution

### 1. Introduction

According to classical thermodynamics, Carnot coefficient of performance is the highest performance parameter of any system having reversible processes while practical systems are irreversible and they cannot achieve Carnot COP. Thus, Carnot COP does not have practical utility. Therefore, it is necessary to establish general performance characteristics with irreversible (real) processes [1–5].

Vapour absorption refrigeration cycles have many advantages over conventional vapour compression refrigeration cycles, like these help to save the primary energy sources and

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lot to keep clean environment. But major problem with these systems is low performance than conventional one, hence there is a great need to optimize these systems [6–12,22,24]. Wijeysundera [19] has used an ideal three-heat-reservoir

can be driven by low grade energy, more importantly, help a

Wijeysundera [19] has used an ideal three-heat-reservoir cycle with constant internal irreversibilities and external heat transfer irreversibilities to model the absorption refrigeration machine of a solar operated absorption cooling system. He obtained analytical expressions for the variation of the entropy transfer with storage tank temperature and the variation of the coefficient of performance (COP) with the cooling capacity of the plant. These expressions give the operating points for the maximum cooling capacity and the maximum COP. The effect of internal and external irreversibilities on the second-law efficiency of the plant is examined. Bautista et al. [21] have analyzed a generalized endoreversible refrigeration cycle characterized by three heat

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Nomenclature			
A COP	surface area of heat exchanger m <sup>2</sup> coefficient of performance of refrigeration cycle	ε	overall COP of vapour absorption refrigeration system
*	(evaporator–condenser assembly)	Greek symbols	
$egin{array}{c} L \ \mathcal{Q} \end{array}$	Lagrangian operator heat transfer to and from the system kJ	$\lambda, \mu$	Langrangian multipliers
R	$(Q_E/\tau)$ cooling load kW	η	efficiency of heat engine (generator–absorber assembly)
$R_{\Delta S}$	internal irreversibility parameter of heat engine cycle	τ	total cycle time of the vapour-refrigeration
$R'_{\Delta S}$	internal irreversibility parameter of refrigeration		$system, = (t_1 + t_2)$
	cycle	Subscripts	
S	entropy $kJ \cdot K^{-1}$	a	absorber
T	temperature K	c	condenser
$t_1$	total cycle time of heat engine cycle,	e	evaporator
	$=(t_g+t_a)$ s	g	generator
$t_2$	total cycle time of refrigeration cycle,	$\overset{\circ}{G}$	heat source/generator thermal reservoir
	$=(t_c+t_e)\ldots\ldots$ s	m	maximum
U	overall heat transfer coefficient . $kW \cdot m^{-2} \cdot K^{-1}$	$\boldsymbol{A}$	absorber thermal reservoirs
W	work output of heat engine cycle or work input	C	condenser thermal reservoir
	to refrigeration cycle kJ	$\boldsymbol{E}$	evaporator thermal reservoir
	•		

sources with finite heat capacities. They have predicted optimal coefficient of performance and the relationship of that with a given rate of refrigeration. The physical influence of the involved finite heat capacities, together with the thermal resistances, on the performance of the refrigeration cycle has been studied. Chen [23] analyzed a two-stage combined refrigeration system taking into account the influence of multi-irreversibilities, such as finite rate heat transfer, heat leak between the heat reservoirs and internal dissipation of the working fluid, on the performance of the refrigeration system. The maximum coefficient of performance with non-zero specific cooling rate is calculated, and other corresponding performance parameters, such as the temperatures of the working fluid in the isothermal processes, the optimal distribution of the heat transfer areas and the power input of the refrigeration system, are determined. Chen [7], has analyzed the vapour-absorption refrigeration system by considering that system is endoreversible (i.e., externally irreversible and internally reversible) and found out a relation between optimal overall coefficient of performance and cooling load. But, he did not consider the effect of internal irreversibility and made optimization only w.r.t. working fluid temperatures and area of heat exchang-

In this paper, a more general optimization of vaporabsorption refrigeration system is established and the optimal analytical bounds on source/sink side heat transfer time, heat engine cycle and refrigeration cycle time and source/sink side heat exchanger areas are determined. System is considered externally as well as internally irreversible. For studying the effect of internal irreversibility, two internal irreversibility parameters ( $R_{\Delta S}$  for generator–absorber

assembly and  $R'_{\Delta S}$  for evaporator–condenser assembly) are introduced in the analysis, defined by second law of thermodynamics following Wu and Kiang [13], Goktun and Ozkaynak [14], Goktun [11], Bhardwaj [25]. The absorptionrefrigeration system is considered as combined cycle, consisting of a heat engine and a refrigeration cycle. Actually temperature of working fluid on generator and condenser side will vary in sub-cooling and superheating regions but in order to simplify the analysis mean effective constant temperature of working fluid are introduced on generator and condenser side following Chen [7], Yan and Chen [9], Bhardwaj [20,25]. This assumption is made to simplify the analysis for studying the external irreversibility of finite heat transfer with surroundings. Applying the concept of finite time thermodynamics, a relation is developed between the overall coefficient of performance and cooling load and hence an optimal bound on overall coefficient of performance corresponding to maximum cooling load is found out. The optimal working regions of the coefficient of performance and cooling load are determined. A detailed parametric study has been carried out and effects of various input parameters have been studied. It is found that the effect of internal irreversibility parameters is more prominent on overall coefficient of performance reduction than the other external irreversibility parameters.

# 2. System description and analysis

Vapour absorption refrigeration system as shown in Fig. 1, has basically four components-generator, absorber, evaporator and condenser. Sub-system consisting of gener-

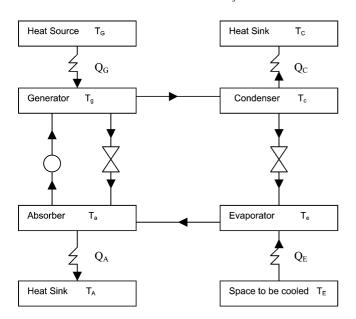


Fig. 1. Schematic diagram of vapour absorption refrigeration system.

ator–absorber assembly is considered as heat engine cycle whereas sub-system consisting of evaporator–condenser assembly is considered as a refrigeration cycle. In actual system, temperature of working fluid on generator and condenser side will vary in sub-cooling and superheating regions but in order to simplify the analysis mean effective constant temperature of working fluid are introduced on generator and condenser side following Davis and Wu [10], Chen [7], Yan and Chen [9], Bhardwaj [25].

In Section 2.1, under the given constraint of given heat input in the generator, an optimization is made w.r.t. source/sink side heat exchanger areas and source/sink side heat transfer time of generator-absorber assembly and an optimal thermal efficiency of heat engine cycle is found out while in Section 2.2, for a given refrigeration effect in the evaporator, an optimization is made w.r.t. source/sink side heat exchanger areas and source/sink side heat transfer time of the evaporator-condenser assembly and an optimal COP expression of refrigeration cycle is found out. Finally, in Section 2.3, the overall coefficient of performance, which is multiplication of the optimal thermal efficiency of generator-absorber assembly and COP of the evaporatorcondenser assembly, is further maximized w.r.t. heat engine and refrigeration cycle time and a general optimal relation between overall coefficient of performance and cooling load is obtained.

#### 2.1. Heat engine cycle analysis

The input heat from heat source to the generator is given by

$$Q_G = U_{\varrho} A_{\varrho} (T_G - T_{\varrho}) t_{\varrho} \tag{1}$$

Similarly, the heat rejection from absorber to the heat sink

$$Q_A = U_a A_a (T_a - T_A) t_a \tag{2}$$

Work output of the generator-absorber assembly is given by

$$W = U_g A_g (T_G - T_g) t_g - U_a A_a (T_a - T_A) t_a$$
 (3)

Second law requires,

$$\Delta S = \frac{U_g A_g (T_G - T_g) t_g}{T_g} - R_{\Delta S} \frac{U_a A_a (T_a - T_A) t_a}{T_a} = 0 \quad (4)$$

Where  $R_{\Delta S}$  is internal irreversibility parameter for heat engine cycle.  $R_{\Delta S}=1$  corresponds to endoreversible system and  $R_{\Delta S}<1$  is for irreversible (real) system. The total heat transfer area of heat engine cycle is  $A_h=A_g+A_a$ . The time taken to complete the heat engine cycle is  $t_1=t_g+t_a$ , since the times required for the two isentropic (expansion/pumping) processes of the cycle are negligibly small relative to the times of isothermal heat addition/rejection following Wu [15,16], Lee and Kim [17], Lee et al. [18].

Under the constraint of given heat input to the generator, the working fluid temperatures may be obtained using Eqs. (1) and (4) and given by:

$$T_g = T_G - \frac{Q_G}{U_g A_g t_g} \tag{5}$$

and

$$T_a = \frac{T_A \left( T_G - \frac{Q_G}{U_g A_g t_g} \right)}{T_G - Q_G \left( \frac{1}{U_g A_g t_g} + \frac{1}{R_{\Delta S} U_a A_a t_a} \right)} \tag{6}$$

On substituting the above value of  $T_a$  in Eq. (2), we have

$$\eta = 1 - \frac{Q_A}{Q_G} = 1 - \frac{T_A/R_{\Delta S}}{(T_G - Q_G(\frac{1}{U_a A_a t_a} + \frac{1}{U_a A_a R_{\Delta S} t_a}))}$$
 (7)

(i) Optimization w.r.t. source/sink side heat transfer area

$$\frac{\partial \eta}{\partial A_g} = 0$$
 gives  $\frac{A_h}{A_g} = 1 + \sqrt{\frac{U_g t_g}{U_a R_{\Delta S} t_a}}$  (8)

and

$$\frac{\partial \eta}{\partial A_a} = 0$$
 gives  $\frac{A_h}{A_a} = 1 + \sqrt{\frac{U_a R_{\Delta S} t_a}{U_g t_g}}$  (9)

Using Eqs. (8) and (9), we have

$$\frac{A_g}{A_a} = \sqrt{\frac{U_a R_{\Delta S} t_a}{U_g t_g}} \tag{10}$$

Substituting the optimal values of  $A_g$  and  $A_a$  into Eq. (7), we have

$$\eta = 1 - \frac{T_A/R_{\Delta S}}{\left(T_G - \frac{Q_G}{A_h} \left(\frac{1}{\sqrt{U_g t_g}} + \frac{1}{\sqrt{U_a R_{\Delta S} t_a}}\right)^2\right)}$$
(11)

(ii) Optimization w.r.t. source/sink side heat transfer time:

$$\frac{\partial \eta}{\partial t_g} = 0 \quad \text{gives} \quad \frac{t_1}{t_g} = 1 + \left(\frac{U_g}{U_a R_{\Delta S}}\right)^{1/3} \tag{12}$$

and

$$\frac{\partial \eta}{\partial t_a} = 0 \quad \text{gives} \quad \frac{t_1}{t_a} = 1 + \left(\frac{U_a R_{\Delta S}}{U_g}\right)^{1/3}$$
 (13)

Using Eqs. (12), (13), we get

$$\frac{t_g}{t_a} = \left(\frac{U_a R_{\Delta S}}{U_g}\right)^{1/3} \tag{14}$$

Now, after substituting these optimal values of the cycle time into Eq. (11), the optimal efficiency of the heat engine is given by

$$\eta = 1 - \frac{T_A/R_{\Delta S}}{T_G - \frac{Q_G}{A_b t_1 K_1}} \tag{15}$$

where

$$A_h = A_g + A_a, \qquad t_1 = t_g + t_a \quad \text{and}$$

$$K_1 = \frac{U_g U_a R_{\Delta S}}{(U_g^{1/3} + (U_a R_{\Delta S})^{1/3})}$$

# 2.2. Refrigeration cycle analysis

The heat extracted from the space to be cooled to the evaporator is given by

$$Q_E = U_e A_e (T_E - T_e) t_e \tag{16}$$

Similarly, heat rejected from condenser to the heat sink

$$Q_C = U_c A_c (T_c - T_C) t_c \tag{17}$$

Work input to the refrigeration cycle is given by

$$W = U_c A_c (T_c - T_C) t_c - U_e A_e (T_E - T_e) t_e$$
 (18)

Second law requires.

$$\Delta S = \frac{U_e A_e (T_E - T_e) t_e}{T_e} - R'_{\Delta S} \frac{U_c A_c (T_c - T_C)}{T_c} = 0 \quad (19)$$

Where  $R'_{\Delta S}$  is internal irreversibility parameter of refrigeration cycle.  $R'_{\Delta S} \leq 1$ , equal to sign holds for endoreversible system and less than sign holds for irreversible (real) system. Similar to Section 2.1, total heat transfer area and cycle time for refrigeration cycle are  $A_r = A_c + A_e$  and  $t_2 = t_c + t_e$ , respectively.

Under the constraint of given cooling effect, the working fluid temperatures may be obtained using Eqs. (16) and (19) and given by:

$$T_e = T_E - \frac{Q_E}{U_e A_e t_e} \tag{20}$$

and

$$T_{c} = \frac{T_{C}(T_{E} - \frac{Q_{E}}{U_{e}A_{e}t_{e}})}{T_{E} - Q_{E}(\frac{1}{U_{e}A_{e}t_{e}} + \frac{1}{R_{A}c}U_{c}A_{c}t_{e}})}$$
(21)

after substituting the value of  $T_c$  into Eq. (17), we have

$$COP = \left[ \frac{T_C / R'_{\Delta S}}{\left( T_E - Q_E \left( \frac{1}{U_e A_e t_e} + \frac{1}{U_c A_c t_c R'_{\Delta S}} \right) \right)} - 1 \right]^{-1}$$
 (22)

(i) Optimization w.r.t. source/sink side heat transfer area

$$\frac{\partial \text{COP}}{\partial A_e} = 0 \quad \text{gives} \quad \frac{A_r}{A_e} = 1 + \sqrt{\frac{U_e t_e}{U_c R'_{\Delta S} t_c}}$$
 (23)

and

$$\frac{\partial \text{COP}}{\partial A_c} = 0 \quad \text{gives} \quad \frac{A_r}{A_c} = 1 + \sqrt{\frac{U_c t_c R'_{\Delta S}}{U_e t_e}}$$
 (24)

using Eqs. (23) and (24), we have

$$\frac{A_c}{A_e} = \sqrt{\frac{U_e t_e}{U_c t_c R'_{\Lambda S}}} \tag{25}$$

On substituting the optimal values of  $A_c$  and  $A_e$  into Eq. (22), we have

$$COP = \left[ \frac{T_C / R'_{\Delta S}}{\left( T_E - \frac{Q_E}{A_r} \left( \frac{1}{\sqrt{U_c t_e}} + \frac{1}{\sqrt{U_c t_e} R'_{\Delta S}} \right)^2 \right)} - 1 \right]^{-1}$$
 (26)

(ii) Optimization w.r.t. source/sink side heat transfer time:

$$\frac{\partial \text{COP}}{\partial t_e} = 0 \quad \text{gives} \quad \frac{t_2}{t_e} = 1 + \left(\frac{U_e}{U_c R'_{\Lambda S}}\right)^{1/3} \tag{27}$$

and

$$\frac{\partial \text{COP}}{\partial t_c} = 0 \quad \text{gives} \quad \frac{t_2}{t_c} = 1 + \left(\frac{U_c R'_{\Delta S}}{U_e}\right)^{1/3} \tag{28}$$

Using Eqs. (27), (28), we have

$$\frac{t_e}{t_c} = \left(\frac{U_c R_{\Delta S}'}{U_e}\right)^{1/3} \tag{29}$$

Substituting the optimal values of cycle time from Eqs. (27), (28) into Eq. (26), the optimal COP of an irreversible refrigeration system operating between reservoirs at temperatures  $T_E$  and  $T_C$  is given by

$$COP = \left[\frac{T_C/R'_{\Delta S}}{\left(T_E - \frac{Q_E}{A_r t_2 K_2}\right)} - 1\right]^{-1}$$
(30)

where

$$A_r = A_c + A_e, t_2 = t_c + t_e \text{and}$$

$$K_2 = \frac{U_e U_c R'_{\Delta S}}{(U_e^{1/3} + (U_c R'_{\Delta S})^{1/3})^3}$$

2.3. Optimization of overall coefficient of performance of the system

Overall coefficient of performance of the vaporabsorption refrigeration system is given by

$$\varepsilon = \left[1 - \frac{T_A/R_{\Delta S}}{T_G - \frac{Q_G}{A_B t_1 K_1}}\right] \left[\frac{T_C/R_{\Delta S}'}{T_E - \frac{Q_E}{A_B t_2 K_2}} - 1\right]^{-1}$$
(31)

Our problem is how to choose  $t_1$  and  $t_2$ , to maximize the overall coefficient of performance  $(\varepsilon)$  of the system. We take overall coefficient of performance  $(\varepsilon)$  as objective function and find its optimum subject to the condition of cooling effect  $Q_E$  and total cycle time  $\tau (= t_1 + t_2)$ ; where,  $t_1$  and  $t_2$  are the heat engine and refrigeration cycle time, respectively) remains unvaried. For such type of constraint optimization

problems, Langrangian undetermined multiplier method is a suitable method, so we introduce the Lagrangian operator

$$L = \varepsilon + \lambda(\tau - t_1 - t_2) \tag{32}$$

Euler-Lagrangian equations

$$\frac{\partial L}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial L}{\partial t_2} = 0$$

give

$$t_{1} = \tau \left[ \sqrt{\frac{T_{A}}{R_{\Delta S}}} \sqrt{K_{2}A_{r}} T_{E} \right.$$

$$+ R \left( \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \frac{1}{\sqrt{K_{1}A_{h}}} - \sqrt{\frac{T_{A}}{R_{\Delta S}}} \frac{1}{\sqrt{K_{2}A_{r}}} \right) \right]$$

$$\times \left[ \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \sqrt{K_{1}A_{h}} \varepsilon T_{G} + \sqrt{\frac{T_{A}}{R_{\Delta S}}} \sqrt{K_{2}A_{r}} T_{E} \right]^{-1}$$

$$t_{2} = \tau \left[ \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \sqrt{K_{1}A_{h}} T_{G} \varepsilon \right.$$

$$- R \left( \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \frac{1}{\sqrt{K_{1}A_{h}}} - \sqrt{\frac{T_{A}}{R_{\Delta S}}} \frac{1}{\sqrt{K_{2}A_{r}}} \right) \right]$$

$$\times \left[ \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \sqrt{K_{1}A_{h}} \varepsilon T_{G} + \sqrt{\frac{T_{A}}{R_{\Delta S}}} \sqrt{K_{2}A_{r}} T_{E} \right]^{-1}$$

$$(34)$$

where cooling load  $R = Q_E/\tau$ .

On substituting these optimal values of  $t_1$  and  $t_2$  into Eq. (31), we have an optimal relation between overall coefficient of performance and cooling load

$$\varepsilon = \left\{ 1 - \frac{T_A}{R_{\Delta S} T_G} - \frac{R}{\varepsilon T_G T_E} \left[ \frac{\varepsilon T_G}{K_2 A_r} + \frac{T_E}{K_1 A_h} \right] \right.$$

$$\left. + \frac{\varepsilon}{\sqrt{K_2 A_r}} \left( \sqrt{\frac{T_A T_C}{R_{\Delta S} R_{\Delta S}'}} \frac{1}{\sqrt{K_1 A_h}} \right.$$

$$\left. - \sqrt{\frac{T_C T_A}{R_{\Delta S}' R_{\Delta S}}} \frac{1}{\sqrt{K_2 A_r}} \right) \right] \right\} \left\{ \frac{T_C}{R_{\Delta S}' T_E} - 1$$

$$\left. + \frac{R}{\varepsilon T_G T_E} \left[ \frac{\varepsilon T_G}{K_2 A_r} + \frac{T_E}{K_1 A_h} + \frac{1}{\sqrt{K_1 A_h}} \right.$$

$$\left. \times \left( \frac{T_C}{R_{\Delta S}' \sqrt{K_1 A_h}} - \sqrt{\frac{T_C T_A}{R_{\Delta S}' R_{\Delta S}}} \frac{1}{\sqrt{K_2 A_r}} \right) \right] \right\}^{-1}$$

$$(35)$$

Rearranging and solving above Eq. (35) for the cooling load (R), we have

$$R = K_1 A_h (\varepsilon_R - \varepsilon) T_G \left( \frac{T_C}{R'_{\Delta S}} - T_E \right)$$

$$\times \left[ T_G \frac{K_1 A_h}{K_2 A_r} (1 + \varepsilon) + T_E (1 + \varepsilon^{-1}) \right]$$

$$+ 2 \sqrt{\frac{T_C}{R'_{\Delta S}}} \sqrt{\frac{T_A}{R_{\Delta S}}} \sqrt{\frac{K_1 A_h}{K_2 A_r}} - \frac{K_1 A_h T_A}{K_2 A_r R_{\Delta S}} - \frac{T_C}{R'_{\Delta S}} \right]^{-1}$$
(36)

where

$$\varepsilon_R = \frac{T_E(\frac{T_A}{R_{\Delta S}} - T_G)}{T_G(T_E - \frac{T_C}{R_{\Delta S}'})}$$

 $\frac{\partial R}{\partial \varepsilon} = 0$  gives

$$\varepsilon_{m} = \left\{ -T_{E} \pm \left[ T_{E}^{2} + \left\{ T_{G} (1 + \varepsilon_{R}) \frac{K_{1} A_{h}}{K_{2} A_{r}} + 2 \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \sqrt{\frac{T_{A}}{R_{\Delta S}}} \sqrt{\frac{K_{1} A_{h}}{K_{2} A_{r}}} \right. \right. \\
\left. + 2 \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \sqrt{\frac{T_{A}}{R_{\Delta S}}} \sqrt{\frac{K_{1} A_{h}}{K_{2} A_{r}}} \right. \\
\left. - \frac{K_{1} A_{h} T_{A}}{K_{2} A_{r} R_{\Delta S}} - \frac{T_{C}}{R_{\Delta S}'} + T_{E} \right\} \varepsilon_{R} T_{E} \right]^{1/2} \right\} \\
\times \left[ T_{G} (1 + \varepsilon_{R}) \frac{K_{1} A_{h}}{K_{2} A_{r}} + 2 \sqrt{\frac{T_{C}}{R_{\Delta S}'}} \sqrt{\frac{T_{A}}{R_{\Delta S}}} \sqrt{\frac{K_{1} A_{h}}{K_{2} A_{r}}} \right. \\
\left. - \frac{K_{1} A_{h} T_{A}}{K_{2} A_{r} R_{\Delta S}} - \frac{T_{C}}{R_{\Delta S}'} + T_{E} \right]^{-1} \tag{37}$$

Maximum cooling load at optimal overall coefficient of performance of the system is

$$R_{m} = K_{1}A_{h}(\varepsilon_{R} - \varepsilon_{m})T_{G}\left(\frac{T_{C}}{R_{\Delta S}'} - T_{E}\right)$$

$$\times \left[T_{G}\frac{K_{1}A_{h}}{K_{2}A_{r}}(1 + \varepsilon_{m})\right]$$

$$+ T_{E}\left(1 + \varepsilon_{m}^{-1}\right) + 2\sqrt{\frac{T_{C}}{R_{\Delta S}'}}\sqrt{\frac{T_{A}}{R_{\Delta S}}}\sqrt{\frac{K_{1}A_{h}}{K_{2}A_{r}}}$$

$$-\frac{K_{1}A_{h}T_{A}}{K_{2}A_{r}R_{\Delta S}} - \frac{T_{C}}{R_{\Delta S}'}\right]$$
(38)

 $\varepsilon_m$  is optimal coefficient of performance corresponding to maximum cooling load  $(R_m)$ . This is also the lowest bound of optimal coefficient of performance for the system as shown in Fig. 2.

## Special cases

(i) Our analysis is more general and Rubin efficiency may be achieved as a special case of our analysis when

$$U_g A_g = U_a A_a = K$$
 (say) and  $R_{\Delta s} = 1.0$ 

then Eq. (7) of Section 2.1, becomes

$$\eta = 1 - \frac{T_A}{T_G - \frac{Q_G}{K\tau}} \tag{39}$$

where

$$\tau = \frac{t_a t_g}{(t_a + t_g)}$$

which is well known Rubin's efficiency of heat engine with given input heat condition. However, with given input heat condition, efficiency cannot be achieved in the form of Curzon–Ahlborn efficiency.

(ii) Similarly for refrigeration cycle analysis, well-known COP expression may be achieved with special case of our analysis when

$$U_e A_e = U_c A_c = K(\text{say})$$
 and  $R'_{\Delta s} = 1.0$ 

then Eq. (22) of Section 2.2, becomes

$$COP = \left[\frac{T_C}{T_F - \frac{Q_E}{E^2}} - 1\right]^{-1} \tag{40}$$

where

$$\tau = \frac{t_e t_c}{(t_e + t_c)}$$

which is well-known COP expression with a constraint of given cooling effect for refrigeration cycle and available in the literature.

However in our analysis, efficiency and COP are further optimized w.r.t. source/sink side heat exchanger area and source/sink side heat transfer time and then finally overall coefficient of performance  $(\varepsilon)$  is optimized w.r.t. the heat engine cycle time  $(t_1)$  and refrigeration cycle time  $(t_2)$ . So, naturally our final expressions would be more general and different from both the Rubin as well as Curzon–Ahlborn expressions.

(iii) When cooling load (R) = 0 and  $R_{\Delta S} = R'_{\Delta S} = 1$ , then Eq. (35) becomes,

$$\varepsilon = \frac{\left(1 - \frac{T_A}{T_G}\right)}{\left(\frac{T_C}{T_E} - 1\right)} = \left(\frac{T_G - T_A}{T_G}\right) \left(\frac{T_C - T_E}{T_E}\right) \tag{41}$$

which is nothing but Carnot COP of a combined cycle operated among the four thermal reservoirs and may be considered as consisting of heat engine cycle working between  $T_G$  and  $T_A$ , and refrigeration cycle working between  $T_C$  and  $T_E$ .

# 3. Results and discussion

It can be seen from Fig. 2 and Eq. (36) that for each value of cooling load there are two coefficient of performance parameters say  $\varepsilon_{\rm I}$  and  $\varepsilon_{\rm II}$  (graphically shown in Fig. 2). Out of these two, one (say  $\varepsilon_{\rm II}$ ) is lower than the  $\varepsilon_m$  and another is (say  $\varepsilon_{\rm I}$ ) higher than  $\varepsilon_m$ . Obviously, higher one  $\varepsilon_{\rm I}$  is optimal value of overall coefficient of performance. The relation between overall coefficient of performance and cool-

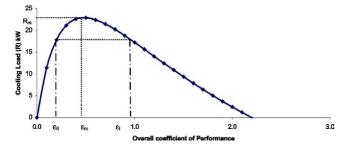


Fig. 2. Effect of cooling load (R) on overall coefficient of performance ( $\varepsilon$ ).

ing load for different values of internal irreversibility parameters  $R_{\Delta S}$  and  $R'_{\Delta S}$  is shown in Figs. 3 and 4. During the variation of any one input parameter, all other parameters are keeping constant as  $T_E = 288 \text{ K}$ ,  $T_G = 393 \text{ K}$ ,  $T_C = 313 \text{ K}, T_A = 318 \text{ K}, R_{\Delta S} = R'_{\Delta S} = 1.0, A_h/A =$  $A_r/A = 0.5$ ,  $U_g = U_a = U_e = U_c = 10.0 \text{ kW} \cdot \text{K}^{-1} \cdot \text{m}^{-2}$ . For these typical input parameters, Eq. (41) gives Carnot COP of the system  $\varepsilon = 2.20$ , Eq. (37) gives optimal overall coefficient of performance for endoreversible system (say,  $R_{\Delta S} = R'_{\Delta S} = 1.0$ )  $\varepsilon_m = 0.48$  whereas using the analytical expression given by Chen [7], the optimal coefficient of performance of an endoreversible system for the same input parameters has been predicted as  $\varepsilon_m = 0.49$ . However, for irreversible system (say,  $R_{\Delta S} = R'_{\Delta S} = 0.95$ ) it may be predicted using Eq. (37) and comes to be  $\varepsilon_m = 0.32$ . Figs. 5–11 show the rate of change in  $\varepsilon_{II}$ ,  $\varepsilon_{I}$  and  $\varepsilon_{m}$ , with input parameters (viz.  $T_E$ ,  $T_G$ ,  $T_A$ ,  $T_C$ ,  $R_{\Delta S}$ ,  $R'_{\Delta S}$ ,  $A_h/A$  and  $A_r/A$ ). Figs. 5–8 show that both  $\varepsilon_I$  and  $\varepsilon_m$  increase rapidly with increasing  $T_E$  than  $T_G$  while these performance parameters decrease more rapidly with increasing  $T_A$  than  $T_C$ . Figs. 9 and 10 show that both performance parameters ( $\varepsilon_{\rm I}$ and  $\varepsilon_m$ ) decrease in prominent way with increasing internal irreversibility (decreasing value of internal irreversibility parameters  $R_{\Delta S}$  and  $R'_{\Delta S}$ ), than external irreversibility parameters. With increasing internal irreversibility ( $R_{\Delta S}$  and  $R'_{\Lambda S}$  is going down from 1.0 to fraction value say 0.91) optimal overall optimal coefficient of performance  $\varepsilon_{\rm I}$  decrease about 61% and 57% while decreasing rate of  $\varepsilon_m$  is about 23% and 38%, with  $R_{\Delta S}$  and  $R'_{\Delta S}$ , respectively. Obviously, internal irreversibility is dominant factor for overall

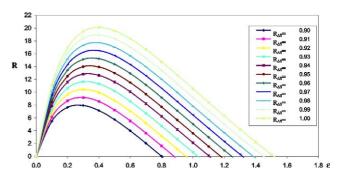


Fig. 3. Cooling load (R) vs. overall coefficient of performance ( $\varepsilon$ ), for varying  $R_{\Delta S}$ .

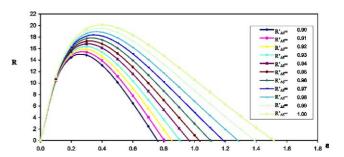


Fig. 4. Cooling load (R) vs. overall coefficient of performance ( $\varepsilon$ ), for varying  $R'_{\Lambda S}$ .

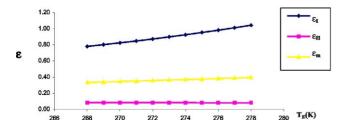


Fig. 5. Effect of  $T_E$  on overall coefficient of performance  $(\varepsilon)$ .

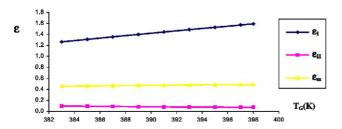


Fig. 6. Effect of  $T_G$  on overall coefficient of performance  $(\varepsilon)$ .

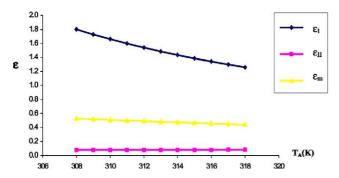


Fig. 7. Effect of  $T_A$  on overall coefficient of performance  $(\varepsilon)$ .

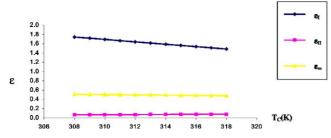


Fig. 8. Effect of  $T_C$  on overall coefficient of performance  $(\varepsilon)$ .

coefficient of performance reduction, but out of these two parameters  $(R_{\Delta S} \text{ and } R'_{\Delta S})$ ,  $R_{\Delta S}$  is more sensitive for performance reduction than  $R'_{\Delta S}$ .

Fig. 11 shows, how distribution of heat exchanger areas affect the performance? Study shows that to keep high overall coefficient of performance, area distribution on the side of evaporator—condenser assembly should be more than that of distributed on generator—absorber side.

Fig. 12 shows the effect of cooling load on overall coefficient of performance. If cooling load is zero, expression of overall coefficient of performance given by Eq. (35) reduces to Carnot COP and with increasing cooling load, performance decreases and has lower value than Carnot COP.

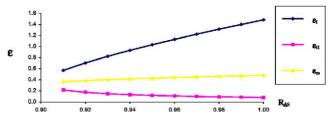


Fig. 9. Effect of  $R_{\Delta S}$  on overall coefficient of performance  $(\varepsilon)$ .

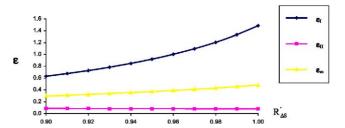


Fig. 10. Effect of  $R'_{\Delta S}$  on overall coefficient of performance  $(\varepsilon)$ .

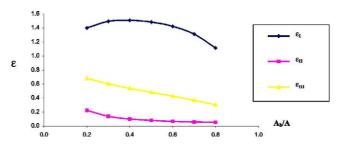


Fig. 11. Effect of  $A_h/A$  on overall coefficient of performance  $(\varepsilon)$ .

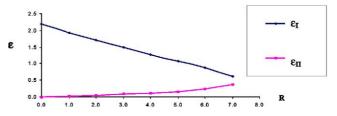


Fig. 12. Effect of cooling load (R) on overall coefficient of performance  $(\varepsilon)$ .

# 4. Conclusions

Vapour absorption refrigeration system is considered as consists of an irreversible heat engine and an irreversible refrigerator. Optimization is made w.r.t. source/sink side heat exchanger areas, source/sink side heat transfer time and heat engine cycle and refrigeration cycle time. An analytical optimal relation between overall coefficient of performance and cooling load is found out. A detailed parametric study has been made and the effect of various input parameters is shown in the form of graphical Figs. 2–12. Parametric study shows that internal irreversibility has more prominent effect on overall coefficient of performance reduction than external irreversibility. It is also found that internal irreversibility parameter of generator—absorber assembly is more sensitive for overall coefficient of performance reduction than the internal

irreversibility parameter of evaporator–condenser assembly. Eq. (37) provides an optimal overall coefficient of performance corresponding to maximum cooling load as well as lower limit of optimal bound on overall coefficient of performance.

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